# RESEARCHES New Correlation of Oil Compressibility at Pressures Below Bubble Point For Iraqi Crude Oils Omar F. Hassan and Dhefaf J. Sadiq , Department of Petroleum Engineering , College of Engineering, University of Baghdad

## ABSTRACT

Oil compressibility represents a significant character in reservoir simulation, design of surface facilities and the analysis of well tests, specifically for systems below the bubble point pressure.

Oil compressibility is not directly measured in the laboratory. It is usually gained indirectly from experimental data recorded in PVT reports. The relative volume from the flash test is used to calculate oil compressibility at pressures above the bubble point pressure. At pressures below the bubble point, the reservoir behavior is simulated by the differential liberation test. The solution gas-oil ratio and the oil formation volume factor from the differential liberation test are employed in the estimation of oil compressibility at pressures below the bubble point pressure This paper purposes new correlation for calculating isothermal oil compressibility coefficient at and below bubble point pressure. The formulation of oil compressibility correlation is very difficult as it depends on many variables. This property is a function of many variables such as bubble point pressure, reservoir pressure, reservoir temperature, solution gas-oil ratio, oil formation volume factor, stock-tank oil gravity, specific gravity of gas and gas formation volume factor.

Standing's (1), McCain et al. (2) and Al-Jarri's (3) correlations were submitted for testing their validity to evaluate their performance with Iraqi crude oils and to compare their results with the results of the new correlation.

The achievement of the new correlations has been done using two hundreds and nine data points from twenty PVT tests that were collected from Southern Iraqi fields. The evaluation of the previous correlations has achieved with graphical and statistical methods. These checking methods show a poor agreement between the observed and the calculated values. The checking methods (graphical and statistical) explain that the new correlation that was achieved in this paper is suitable to calculate oil compressibility below bubble point pressure for Iraqi crude oils.

#### الخلاصة

انضغاطية النفط تمثل خاصية مهمة في حسابات هندسة المكامن و هندسة انتاج النفط خصوصا للحالات التي تكون تحت ضغط الفقاعة. هذه الخاصية لا تقاس بشكل مباشر في المختبر لكن يمكن الحصول عليها بصورة غير مباشرة من البيانات المختبرية لتقارير الخواص الفيزياوية و الثرموديناميكية للنفوط المكمنية. الحجم النسبي المقاس خلال تجارب العزل الوميضي يستعمل لقياس انضغاطية النفط عند ضغوط اعلى من ضغط الفقاعة بينما تحسب قيم هذه الخاصية تحت ضغط الفقاعة بالاعتماد على القيم المقاسة مختبريا لكل من معامل التكوين الحجمي للنفط و نسبة الغاز المذاب في النفط المقاسة خلال تجارب التحرر التفاضلي.

البحث يقترح معادلة تجريبية جديدة لحساب انضغاطية النفط تحت ضغط الفقاعة بالاعتماد على الطرق الرياضية و الاحصائية. ان عملية الصياغة لهذه المعادلة تعتبر معقدة جدا لان انضغاطية النفط (المتغير المعتمد) تعتمد على عدة متغيرات (المتغيرات المستقلة) و هي درجة حرارة المكمن و الضغط المكمني و ضغط الفقاعة و نسبة الغاز المذاب في النفط و معامل التكوين الحجمي للنفط و معامل التكوين الحجمي للغاز و الكثافة النسبية للغاز و كثافة النفط في الظروف القياسية.

المراجعة للادبيات العلمية لهذا الموضوع وجدت ثلاثة معادلات تجريبية صيغت لحساب انضغاطية النفط عند ضغوط اقل من ضغط الفقاعة و هي كالتالي :

- Standing
- McCain
- Al-Jarri

لذلك استعملت هذه المعادلات التجريبية لمقارنة نتائجها مع نتائج المعادلة التجريبية التي اعدت خلال هذا البحث بالاعتماد على البيانات العراقية. المعادلة الجديدة انجزت باستخدام مائتين و تسعين قياس مختبري استخلصت من عشرين تقرير مختبري لقياس الخواص الفيزياوية و الثرموديناميكية للنفوط المكمنية.

التقييم للمعادلات التجريبية السابقة التي تقوم بحساب قيم انضغاطية النفوط تحت ضغط الفقاعة تم من خلال تطبيق طرق الرسم و المعايير الاحصائية و اوضحت هذه الطرق عدم كفاءة هذه المعادلات عند تطبيقها مع النفوط العراقية و عند تطبيق نفس طرق التقييم مع المعادلة التجريبية الجديدة تبين دقة النتائج المستحصلة منها و التي كانت ادق من النتائج المستحصلة من المعادلات التجريبية المنشورة سابقا.

## 2. Introduction

(1)

The isothermal compressibility coefficient is defined as the rate of change in volume of a substance with respect to pressure increase per unit volume. Mathematically, the isothermal compressibility of a substance is defined by the following expression:

$$c = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)$$

Oil isothermal compressibility coefficients are required, among other characters, in all solutions of transient fluid-flow problem. Accurate evaluation of this character will be valuable for the interpretation of well test analysis and other reservoir performance calculations such as estimate reserve and reservoir simulation.

For a crude oil system, the isothermal compressibility coefficient of the oil phase is categorized to the following two types based on reservoir pressure:

2. Below the bubble-point pressure, the solution gas is liberated with decreasing reservoir pressure or redissolved with increasing the pressure. The change of the oil volume as a result of changing the gas solubility must be considered when determining the isothermal compressibility coefficient. The oil compressibility in this case is termed saturated isothermal compressibility coefficient.

At pressures below the bubble-point pressure, the isothermal compressibility coefficient of the oil must be modified to account for the shrinkage associated with the liberation of the solution gas with decreasing reservoir pressure or swelling of the oil with repressurizing

- 1. At reservoir pressure which is greater than or equal to the bubble-point pressure (  $P \ge P_b$ ), the crude oil exists as a single phase with all its dissolved gas solution. The still in isothermal compressibility coefficient of the oil phase above the bubble point reflects the changes in the volume associated with oil expansion or compression of the single-phase oil with changing the reservoir The pressure. oil compressibility in this case is termed undersaturated isothermal compressibility coefficient.
- 2. volume as the result of changing the gas solubility must be considered when determining the isothermal compressibility coefficient of the oil compressibility. Below the bubble-point pressure is defined by the following expression <sup>(4) (5)</sup>:

the reservoir and redissolving the solution gas. These changes in the oil

$$c_{o} = -\frac{1}{B_{o}} \left[ \frac{\partial B_{o}}{\partial P} \right]_{T} + \frac{B_{g}}{B_{o}} \left[ \frac{\partial R_{s}}{\partial P} \right]_{T}$$
(2)

But, the independent variables of equation (2) are not available for all wells and oil compressibility must be calculated from correlations.

Oil compressibility values that are obtained using equation (2) will represent the observed values of this property.

Very few researches have been made regarding correlations for oil compressibility at pressures below the bubble point pressure.

Analytically, Standing's correlations <sup>(1)</sup> for Rs and Bo can be

differentiated with respect to the pressure, p, to give

$$c_{o} = \frac{-R_{s}}{B_{o}(0.83 P + 21.75)} \left[ 0.000144 \sqrt{\frac{\gamma_{g}}{\gamma_{o}}} \left[ R_{s} \sqrt{\frac{\gamma_{g}}{\gamma_{o}}} + 1.25 (T - 460) \right]^{0.2} - B_{g} \right]$$
(3)

McCain et. al. <sup>(2)</sup> (1988) correlated the oil compressibility with pressure, oil API gravity; gas solubility at the bubble point, and temperature. Their proposed correlation has the following form:

$$c_o = e^A \tag{4}$$

where the correlating parameter A is given by the following expression:

$$A = -7.633 - 1.497 \ln(P) + 1.115 \ln(T) + 0.533 \ln(API) + 0.184 \ln(R_{sp})$$
 (5)

The authors suggested that the accuracy of equation (4) can be substantially improved if the bubble-point pressure is known. They improved the correlating parameter, A, by including the bubble-point pressure, P<sub>b</sub>, as one of the parameters in the preceding equation, to give  $A = -7.573 - 1.45 \ln(P) - 0.383 \ln(P_b) + 1.402 \ln(T) + 0.256 \ln(ABL)$ 

Al-Jarri<sup>(3)</sup> (1990) suggested correlation for estimating isothermal oil compressibility at and below bubble point pressures. This correlation was derived using oils that were collected from Middle East and North America. It was derived as a function of pressure, bubble point pressure, gas relative density, solution gas-oil ratio at bubble point pressure, and reservoir temperature. His correlation has the following form

$$C_{o} = e^{-7.626} P^{-1.726} P_{b}^{-0.8} R_{sb}^{0.659} T^{1.302} \gamma_{g}^{-0.304} e^{0.0002(P-P_{b})}$$
(7)

# The Estimation of the Observed Oil Compressibility at Pressures Below Bubble Point

The estimation process of this property has been done using equation (2) but its difficulty is the calculation of the two derivatives. The determination of these derivatives was done by two techniques:

#### 1. Conventional Technique

This technique directly depends on the laboratory values of solution gas-oil ratio, oil formation volume factor and pressure. The following equations are used to calculate the values of derivatives

$$\frac{dR_s}{dP} = \frac{R_{s1} - R_{s2}}{P_1 - P_2}$$
(8)  
$$\frac{dB_o}{dP} = \frac{B_{o1} - B_{o2}}{P_1 - P_2}$$
(9)

#### 2. New Technique

In this technique, the correlations of solution gas-oil ratio and oil formation volume factor are derived with respect to pressure to estimate their derivatives that will be used to calculated oil compressibility values below bubble oil pressure.

In this paper, two correlations in Sixth and Seventh references of this paper were used to find the required derivatives. The equations (10) and (11) were already achieved using Iraqi crude oils in previous researches (Hassan and Sadig). The equations and (11a) represent the (10a) derivatives of solution gas-oil ratio and oil formation volume factor, respectively.

$$R_{s} = \begin{pmatrix} A_{0} & P_{b}^{A_{1}} & \gamma_{g}^{A_{2}} & T^{A_{3}} & API^{A_{4}} & R_{sb}^{A_{5}} \end{pmatrix} P^{\left[ \left( A_{6} & P_{b}^{A_{7}} & \gamma_{g}^{A_{8}} & T^{A_{9}} & A \right) \right]}$$
(10)

Where  $A_0$ ,  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$ ,  $A_5$ ,  $A_6$ ,  $A_7$ ,  $A_8$ ,  $A_9$ ,  $A_{10}$ , and  $A_{11}$  represent the constants of equation (10a)

$$B_{o} = a_{1} P^{a_{2}} T^{a_{3}} API^{a_{4}} \gamma_{g}^{a_{5}} + a_{6}$$
(11)

$$\frac{dB_{o}}{dP} = a_{1} a_{2} P^{(a_{2}-1)} T^{a_{3}} API^{a_{4}} \gamma_{g}^{a_{5}}$$
(11a)

Where  $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_4$ , and  $a_5$  represent the constants of equation (11a)

The observed values were determined using the two techniques for calculating the derivatives in Equation (2).

The values of constants of equations (10) and (11) were calculated in Sixth and Seventh references.

#### **Formulation of New Correlation**

The first step for formulating of a correlation is regression analysis that gives information on the relationship between a response variable and one or more independent variables to the extent that information is contained in the data. The goal of regression analysis is to express the response variable as a function of the predictor variables. Duality of fit and accuracy of conclusion depend on the data used <sup>(8)</sup>.

Oil compressibility characterizes response variable with bubble point pressure, pressure, reservoir temperature, stock-tank oil gravity and specific gravity of gas  $\operatorname{API}^{A_{10}} \operatorname{Period}^{A_{10}}$  esent the independent variables.

Observed oil compressibility values that were used in regression analysis have been obtained by applying new technique.

Many forms of suggested relationships have been tried for getting the best empirical equation corresponds that to minimum difference between the observed and calculated values. Statistical and graphical tests are employed on these forms to gain the following relationship.

 $c_{o} = e^{a_{1}} API^{a_{2}} \gamma_{g}^{a_{3}} P_{b}^{a_{4}} T^{a_{5}} e^{(a_{6}(P_{b}-P))} + a_{7} 10^{(a_{8}P)} + a_{9}$ (12)

Table (1) shows values of the constants of equation (12) that were obtained using regression analysis.

# Evaluation of the Correlations <u>1-Statistical Analysis</u>

This section has been divided into two parts: the first took the observed values using conventional technique while the second depended on new technique to obtain observed values.

Average percent relative error, average percent of absolute relative error, loss function and standard deviation were computed for each correlation. Two hundreds and nine data points from twenty PVT reports have been subjected to evaluation process for each correlation.

#### First Part

Table (2) shows the statistical criteria for the correlations. The new correlation has best values of average percent relative error, average percent of absolute relative error, loss

function and standard deviation while McCain's correlation yield the worst values of statistical criteria. As shown in Table (2), the new correlation presented good performance whereas the published correlations were very poor.

## Second Part

Table (3) also gives the statistical criteria for the same correlations. The new correlation has the best statistical criteria. The statistical criteria of the New correlation in the second part is better than in the first part because the new technique for calculating the observed oil compressibility values simulated the behavior of each of solution gasoil ratio and oil formation volume factor with respect to pressure better than the conventional technique.

This part demonstrates that the New correlation has the validity to apply with Iraqi oils while the published correlations still weak.

# 2-Graphical Test

Oil sample that which is used to create New correlation has been subjected to the graphical test. Figures (1) through (4) show the degree of matching between the observed and calculated values. Standing's correlation has about 30% error while McCain's correlation and Al-Jarri's correlation cannot match with the observed values as shown in figures (2) through (4). The new correlation explains good agreement between the actual and predicted values as shown in Figure (1).

#### **Conclusions**

1-The accuracy of the published correlations was not adequate to fit the observed values of oil compressibility for Iraqi crude oils below bubble point pressures as they have poor statistical results and bad cross plots.

2- The New correlation can predict oil compressibility for Iraqi crude oils below bubble point pressures with good matching.

3- The new correlation and the new technique can be applied without using PVT reports.

4- The performance of the new technique to calculate the derivates of solution gas-oil ratio and oil formation volume factor with respect to pressure is better than the performance of the conventional technique.

## <u>Nomenclature</u>

API : API gravity

Bo : Oil formation volume factor, bbl/STB

Bg : Gas formation volume factor, bbl /scf

 $C_{o}$  : isothermal oil compressibility coefficient, Psi<sup>-1</sup>

P : Pressure, psig

- P<sub>b</sub>: Bubble Point Pressure, psig
- R<sub>s</sub>: Solution gas-oil ratio, scf/STB
- T : Reservoir Temperature, <sup>o</sup>F
- $\gamma_{g}$ : Specific gravity of gas
- $\gamma_{o}$ : Specific gravity of oil

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	The Constant	Value
	$\mathbf{a}_1$	81.75447
	<b>a</b> <sub>2</sub>	1.459646
	a3	1.713428
	a4	-11.5505
	a5	-1.53784
-	a <sub>6</sub>	0.003822
	<b>a</b> <sub>7</sub>	12.66838
	<b>a</b> <sub>8</sub>	$-2.837 \times 10^{-8}$
	ag	-12.66538

# Table (1)

## Table (2)

	New	Standing	McCain	Al-Jarri
Average percent relative error	-24.851	69.63	-95.987	-94.882
Average percent of absolute relative error	26.6535	69.63	95.987	95.286
Loss function	0.001896739	0.0230	0.0217	0.0214
Standard deviation, %	34.533	83.563	96.327	95.663

	New	Standing	McCain	Al-Jarri	
Average percent relative error	4.779	42.657	-96.355	-94.002	
Average percent of absolute relative error	10.414	46.278	96.355	96.607	
Loss function	5.5×10 <sup>-6</sup>	8.6×10-5	8.8×10 <sup>-5</sup>	1.2×10 <sup>-4</sup>	
Standard deviation, %	15.796	48.917	96.635	3307.947	





**Figure (1) Cross Plot for the New Correlation** 







Figure (4) Cross Plot for Al-Jarri's Correlation